Local and global information and equations with left and right invertible operators in the free Fock space

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Abstract

The subject of the work is short description of huge amount of n-point information (n-pi) about the system. Methods for solving equations that satisfy such information are considered. Possible interpretation of left and right invertible operators appearing in these equations is also proposed. For local information, the creation and annihilation operators satisfying the Cuntz relations are introduced. It is also introduced the vector describing the local vacuum which completes the equation for n-pi with a global information. An important components of the work are examples of local operator-valued functions.

Contents

1 Introduction

Quotations:

"Recently there have been ambitious attempts to ground all of physics in information; in other words, to treat the universe as a gigantic informational or computational process (Frieden, 1998). An early project of this type is Wheeler's "It from bit' proposal (Barrow, Davies & Harper, 2003). We might call this 'level inversion' since information is normally regarded as a higher-level concept than, say, particles." from P.Davies article - "The physics of downward causation"

The letters and words arranged with them, numbers, variables, and various symbols used in science - are used to describe the environment and ourselves. We are talking about variables when they may take different values, but about the time-dependent functions when the values that take variables are ordered, tracked over time. In the case of large amounts of data concerning various aspects of the described system, it is convenient to introduce the generating vector whose components are the data. Moreover, often, the components are functions not only depending on time, but from other variables and so infinite, often uncountable amount of data, information about the system, one must be able to describe in a compact way not only to control them, but to be able to pass them or allow to them access of others.

The information contained in text such as - official letter or a novel or a description of physical or mathematical theory - can be described using the appropriate function $\varphi(t,\vec{x})$ where t - is a given time, and $\overrightarrow{x}$−a point in space or place in the book. In the latter case, the value of function φ is the given letter of the alphabet $\mathcal{A}$, punctuation or spacing between words. In the case of a physical system, such as electromagnetic field, φ is a multi-functions $\varphi_i(t,\vec{x}); i = 1,...,N$ describing the components of the field, which is further, to save the recording, will be denoted by $\varphi(\tilde{x})$.

Often, too detailed information $\varphi(\tilde{x})$ describing the system (gas in the vessel, book, picture) is not available or indicated, and then we are satisfied by its averaged or smoothed quantity $< \varphi(\tilde{x}) >$: In other words, we are satisfied by $\varphi(\tilde{x}) \rightarrow < \varphi(\tilde{x}) >$. In fact, such process is carried out by our senses, instruments or our theories, which are tailored to our manual and intellectual capabilities. However, it is something in human nature that we would like to know what is behind the world only accessible to our senses and instruments and therefore so often what is really $< \varphi(\tilde{x}) >$ is denoted by $\varphi(\tilde{x})$ which need to be discovered. Here it is worth mentioning the possibility of a completely crazy idea that $\tilde{x}$ may not have nothing to do with time and space, and appropriate averaging only may have such relationship, see [13], page 413.

It is not excluded that even at the classical level, there is a whole hierarchy

of descriptions in which each level is obtained by proper filtration related to available instruments and theories. In other words, the increasingly lower levels need not to be associated with more subtle molecular structure of matter but rather with very sensitive, ill posed description of the problem. Just as there is no point to make ever finer triangulation of the measured surface, where individual measurements are subject to a specific error, so it does not make sense to look for increasingly accurate solutions to the Navier-Stocke equations if we do not know the exact initial and boundary conditions.

And now there is really a new thing that requires a new paragraph: the value of φ at point $\tilde{x}$, which could mean some local feature of the system under test, such as the position of the i-th item, and especially the change of φ, depends on other system elements. And because this relationship is generally weakens with distance, non-linearity appears here. The equations describing the system are generally nonlinear. And what does this mean for the smoothed or averaged quantities? It turns out that this leads to the need to consider besides quantity $<\varphi(\tilde{x})>$ - averaged products - $<\varphi(\tilde{x}_1)...\varphi(\tilde{x}_n)>; n=1,...,\infty$, so to get a complete, infinite set of equations, which in previous work we called - *n-point (local) information (n-pi) of the system*.

The amount of local information about some physical systems or other complex systems, such as economic, can not be written as a column or row, used in the case of vectors with a finite or countable number of components. Assuming that each of the n-pi is the coefficient standing at the appropriate base vectors: $|\tilde{x}_1,\cdots,\tilde{x}_n>\equiv|\tilde{x}_{(n)}>$ (Dirac notation), we use the symbols of the sum and integral to introduce the vector $|$ V$>$ generating all these information:

$$|V>=<>|0>+\sum_{n=1}^{\infty}\int d\tilde{x}_{(n)}<\varphi(\tilde{x}_1)...\varphi(\tilde{x}_n)>|\tilde{x}_{(n)}> \tag{1}$$

where coefficient standing at the vector $|0>$ does not contain any local information about the system.

The use of one quantity, which is a vector $|V>$, instead of an infinite number of n-pi, is a convenient simplification, and therefore the operations referred to n-pi we also try to describe (express) with a generating vector $|$ V$>$. In this and previous works, we try to express them by means of creation and annihilation operators which satisfy the Cuntz relations,5. This leads to a very simple structure for orthogonal basis vectors $|\tilde{x}_{(n)}>$, see Eq.3, which leads to a simple relations between the generating vector $|$ V$>$ and n-pi:

$$<\tilde{x}_{(n)}|V>=<\varphi(\tilde{x}_1)...\varphi(\tilde{x}_n)> \tag{2}$$

The use of generating vectors instead of generating functionals (fg), see [Rzewuski 1969], does not require assumptions about the convergence of the respective sums. We will also assume that the n-pi $<\varphi(\tilde{x}_1)...\varphi(\tilde{x}_n)>$ are the usual functions, rather than generalized functions and, despite use of the symbol of the integral, $\int$, the value of multicomponent symbols $\tilde{x}$ are discrete, simply because to remember that we do not exclude the continuous case. In other words, we assume some regularization of considered theory.

The set of vectors 1 constructed with the help operators fulfilling the Cuntz relations (5) is called the *free (super, full) Fock space*. In this space the equation for the generating vector |V> is constructed by means of one-side invertible operators with explicitly constructed inverse operators. This allows us to construct a suitable projection operators by which we express the general solution. But a more important and difficult is a choice of physical solutions which we try to obtain by additional assumptions expressed, for example, by the perturbation or closure principles. It is worth noting that the free Fock space allows us to describe the exact and averaged differential equations, both in the commuting and noncommuting cases.

Novelty of the submitted work, which can be read independently from others author papers, is the in-depth understanding of the developing formalism, in particular the role of one-side invertible operators and the introduction of a new class of nonpolynomial operators describing dynamics of the system, for which is given an appropriate perturbation calculations. And although still not laid the appropriate equations describing the dynamics of the formation of novels or other products of humanism, the fact that the even novel events can be described using the appropriate field $\varphi(\tilde{x})$ and that at the perception of images, writing or reading novel in some sense - we use filtration $< \varphi(\tilde{x}) >$, leads us to believe that in this ever-changing, fluid reality, (Zygmund Bauman), situation may change. Notwithstanding this, the equations that we are considering are already in operation in many fields of science and technology.

In a certain sense, by introducing the local and global entities, we touched on the philosophical problem - the top-down causation, see [12], getting a new look on spooky forces exerted by wholes upon their components! In fact, the global variables are integrated in some way fields. They can be trated as parameters of theory. One can derive equations upon them and then it would means that our ignorance about the system is not arbityrary but is limited by solutions to these equations. To such conclusion we should come if we want to keep in mind other useful computation assumptions like the perturbation or closure principles. If in addition it would appear that this is the only way of computation success, we would have to assume that only certain ways of filtering leads to new information about the system. In this sense we can say that our ignorance about the system is quantized or rather limited!

2 Operators that create and annihilate (local) information and vectors that represent "nothingness"

In many cases the components of a vector have direct physical interpretation. For example, the components of the radius vector can be interpreted as projections of the position of the material point on the appropriate base vectors. In quantum mechanics (QM) the physical interpretation have got components as well as base vectors, which are eigenvectors of appropriate Hermitian operators

like the Hamilton operator or the momentum operator. **To descsibe local information about the system we do not need Hermitian operators**: That what we need are one-side invertible operators. Among these operators are particularly important cration and annihilation operators. With the help of these operators and the "vacuum" vector $|0>$ we create base vectors:

$$|\tilde{x}_{(n)}>=\hat{\eta}^{\star}(\tilde{x}_1)\cdots\hat{\eta}^{\star}(\tilde{x}_n)|0> \tag{3}$$

Here, from definition, the operator $\hat{\eta}^{\star}(\tilde{x})$ *creates a local information* about the system at the space-time point $\tilde{x}$. The operator which annihilates this information is denoted by $\hat{\eta}(\tilde{x})$. We will assume that

$$\hat{\eta}(\tilde{x})\hat{\eta}^{\star}(\tilde{x})=\hat{I} \tag{4}$$

where $\hat{I}$ is the unit operator in the space of generating vectors. From the one side-invertible point of view, the property (4) allows to call the operator $\hat{\eta}(\tilde{x})$ a right invertible operator, or, more rarely - a *derivative*, and the operator $\hat{\eta}^{\star}(\tilde{x})$ - a left invertible, or, a right invers to the operator $\hat{\eta}(\tilde{x})$.

Additional assumption:

$$\hat{\eta}(\tilde{x})\hat{\eta}^{\star}(\tilde{y})=\delta(\tilde{x}-\tilde{y})\cdot\hat{I} \tag{5}$$

leads to equality:

$$\hat{\eta}(\tilde{x})|\tilde{x}_1,\tilde{x}_2,...,\tilde{x}_n>=\delta(\tilde{x}-\tilde{x}_1)|\tilde{x}_2,...,\tilde{x}_n> \tag{6}$$

which, for the operator $\hat{\eta}(\tilde{x})$, justify the name - the *annihilation operator*. From (6) we also have:

$$\int d\tilde{x}\hat{\eta}(\tilde{x})\hat{\eta}^{\star}(\tilde{y})=\int d\tilde{y}\hat{\eta}(\tilde{x})\hat{\eta}^{\star}(\tilde{y})=\hat{I} \tag{7}$$

Using base vectors (3) and (5), one can show that

$$\int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\hat{\eta}(\tilde{x})=\hat{I} \tag{8}$$

Relations (5) in physical literature are called the *Cuntz relations*. To be in agreement with the usual restrictions imposed upon the vectors $|0>$ and $<0|$:

$$\hat{\eta}(\tilde{x})|0>=0,\ <0|\hat{\eta}^{\star}(\tilde{x})=0 \tag{9}$$

where $<0|$ means a conjugate vector to the vector $|0>$ on which operators act from their left hand side, we modify Eq.8 as follows:

$$\int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\hat{\eta}(\tilde{x})=\hat{I}-|0><0|\Leftrightarrow\hat{I}=\int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})\hat{\eta}(\tilde{x})+|0><0| \tag{10}$$

with restriction $<0|0>=1$.

Eqs (9) can be interpreted in the following way: the operator $\hat{\eta}(\tilde{x})$ acting on the vacuum vector $|0>$, which is related only to global information about the system, see [?], destroys this information completely and this fact is represented by the zero vectors $\vec{0} \equiv 0$ or $\overleftarrow{0} \equiv 0$. The same applies to the operators $\hat{\eta}^{\star}(\tilde{y})$ acting on the left. These two kind of vectors represent *nothingness* (no local information about the system).

With Cuntz relations (5) and assumptions (9), which means that the vectors 3 are orthonormal, it is easy to prove:

$$< 0|\hat{\eta}(\tilde{x}_1)\cdots\hat{\eta}(\tilde{x}_n)|V >=< \varphi(\tilde{x}_1)...\varphi(\tilde{x}_n) > \tag{11}$$

and that

$$< 0|V >=<> \tag{12}$$

The following operators called projectors are also useful:

$$\hat{P}_n = \int d\tilde{x}_{(n)}\hat{\eta}^{\star}(\tilde{x}_1)\cdots\hat{\eta}^{\star}(\tilde{x}_n)|0 >< 0|\hat{\eta}(\tilde{x}_n)\cdots\hat{\eta}(\tilde{x}_1);\ \hat{P}_0 = |0 >< 0| \tag{13}$$

Now we have

$$|V >=<> |0 > + \sum_{n=1}\hat{P}_n|V > \tag{14}$$

3 Equations for n-pi. An important modification

We postulate the following linear equations for the n-pi $< \varphi(\tilde{x}_1)...\varphi(\tilde{x}_n) >$which by means of the generating vector |V> can be described in the following way:

$$\left(\hat{L} + \lambda\hat{N} + \hat{G}\right)|V >= \hat{P}_0\hat{L}|V > +\lambda\hat{P}_0\hat{N}|V >\equiv |0 >_{info} \tag{15}$$

where all operators $\hat{L}, \hat{N}, \hat{G}$ are linear operators in the space of generating vectors (1) (only here we do not use the name - Fock space - to emphasise that we do not assume a norm space). Subsequent operators called here *interaction operators* can be related to subsequent terms in the equation:

$$L[\tilde{x};\varphi] + \lambda N[\tilde{x};\varphi] + G(\tilde{x}) = 0 \tag{16}$$

for the field φ, where L depends in a linear way on the field φ, and N in a nonlinear way, see below. G is a given function. We call (16) the field equation. As an example of such equation can be the Navier-Stock equations. There is simple relation of functionals L, N and function G with operators $\hat{L}, \hat{N}, \hat{G}$, see e.g. [?]. The most general characteristic of these operators is that the operator

$$\hat{L} = \int \hat{\eta}^{\star}(\tilde{x})L(\tilde{x},\tilde{y})\hat{\eta}(\tilde{y})d\tilde{x}d\tilde{y} + \hat{P}_0 \tag{17}$$

is a diagonal operator with respect to the projectors $\hat{P}_n$:

$$\hat{P}_n \hat{L} = \hat{L} \hat{P}_n \tag{18}$$

where n=0,1,... Also, the operator

$$\hat{N} = \int \hat{\eta}^\star(\tilde{x}) N[\tilde{x};\hat{\eta}] d\tilde{x} + \hat{P}_0 \hat{N} \tag{19}$$

is an upper triangular operator:

$$\hat{P}_n \hat{N} = \sum_{n<m} \hat{P}_n \hat{N} \hat{P}_m \tag{20}$$

where n=0,1,..., and the operator

$$\hat{G} = \int \hat{\eta}^\star(\tilde{x}) G(\tilde{x}) d\tilde{x} \tag{21}$$

is a lower triangular operator:

$$\hat{P}_n \hat{G} = \hat{P}_n \hat{G} \hat{P}_{n-1} \tag{22}$$

for n=1,2,..., and, for n=0, $\hat{P}_0 \hat{G} = 0$. A more general form of the lower triangular operator used in fact in quantum theories has the following projection properties:

$$\hat{P}_n \hat{G} = \sum_{m<n} \hat{P}_n \hat{G} \hat{P}_m \tag{23}$$

They express very deep, qualitative properties of described systems like this that the system is immersed in a given external field, case (21), or that the system is subjecting to certain constraints that are implemented without the participation of reaction forces, case ([2];Sec.4.2).

An important modification

As you can see from the above, Eq.15 is identically satisfied for its projection with the help of the projector $\hat{P}_0$. This situation was caused by the fact that the operators $\hat{L}$ and $\hat{N}$ have been modified by adding to their expressions the terms with projector $\hat{P}_0$, see (17) and (19). In this way we obtained the operators, which in many cases are at least right or left invertible, [3]. Such a modification of Eq.15 does not affect the normal perturbation calculations applied to Eq.15, but can influence other approach, see [3], for which Eq.15 without term $|0>_{info}$ is not complete. These are the mathematical reasons for the emergence of vector $|0>_{info}$ in the right hand side of the Eq.15.

Let us now look at this vector with a more physical point of view: As it is known in many well-known equations of physics in their right hand sides - the source of the fields that describe these equations - appears. For example, see Maxwell's or Poisson's equations. Looking at this spirit on the Eq.15, we can say that the vector $|0>_{info}$ is a source of the vector |V>. In other worde,

the source of local information about the system is the vector containing global information about the system, see also [4] and [3].

It is also not excluded that such modification of equations on the generating vector |V> can be used for rethinking some problems in astrophysics and physics and perhaps to rescue certain useful paradigmas like universality of law of physics, reductionist approach and so on, [9], so useful in previous development of science.

4 Information overload and closure problem

As we know many systems can in principle be described by means of varies functions or fields. Even a novel or a picture can be described in this way. If the system is too complex we have to use a certain method of simplification of its description to get some practical results. One such method is the filtering of information, which we illustrate as follows:

$$\varphi(\tilde{x}) \Rightarrow < \varphi(\tilde{x}) > \tag{24}$$

The process shown in (24) can be realized by omission, deletion or averaging too detailed information. The difficulty that arises here is that the functions φ, not as in the case of books written or painted image, are not known. In many cases we know only equations for these functions like Eqs (16) whose solution is not a simple task and it is not even recommended due to consisting too detailed information. In view of these difficulties in the nineteenth century have emerged the idea to consider the equation for the 1-pi $< \varphi(\tilde{x}) >$. But here there is a new difficulty: the equation for 1-pf usually contains other n-pfs, see Eq.15 and Eq.20, and this difficulty is called the closure problem.

The simplest recipe to solve the closure problem is to reject other n-pi. But such method is justified only for very small value of the coupling constant λ if the generating vector |V> depends analytically on λ. Here comes yet another difficulty, namely the coupling constant λ, which describes the properties of nonlinear interaction of the components of the system, is not generally small. We have here such a situation that the nonlinearity problem at the micro level is transformed into the closure problem at the macro level. We have a paradoxical situation: what is simple, namely the interaction between micro components of the system - leads to a difficult closure problem. Since the nonlinearity in the micro-level is common, because the decrease of interaction between components of the system usually appear together with increase of their distances, therefore the closure problem is common. Hence, different methods of closing equations for n-pi are developed, see [3], [5], [6] and literature there cited.

5 A few examples of operators $\hat{N}$

Although n-pi $< \varphi(\tilde{x}_1)...\varphi(\tilde{x}_n) >$ are permutation symmetric, this does not mean that we must use the symmetric base vectors $|\tilde{x}_{(n)} >$. Hence, instead of

using customarily accepted Heisenberg relations:

$$[\hat{\eta}(\tilde{x}), \hat{\eta}^\star(\tilde{y})]_\mp \propto \delta(\tilde{x}-\tilde{y})\hat{I} \tag{25}$$

which in result lead to permutation symmetric or anti symmetric bases vectors $|\tilde{x}_{(n)}>$, we have used the Cuntz relations (5). However, this has the advantage that the operators appearing in Eq.15 are right or left invertible. For example,

$$\hat{N} = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})\hat{\eta}^2(\tilde{x}) + \hat{P}_0 \int d\tilde{x}\hat{\eta}(\tilde{x})f(\tilde{x}) \tag{26}$$

appearing in the so called the Hurst model in Quantum Field Theory, where f is an arbitrary function, is the right invertible operator with an easy constructed right inverse operator:

$$\hat{N}_R^{-1} = 1/2\int d\tilde{y}\hat{\eta}^\star(\tilde{y})^2\hat{\eta}(\tilde{y}) + 1/2\int d\tilde{y}\hat{\eta}^\star(\tilde{y}) \tag{27}$$

if $\int d\tilde{x}f(\tilde{x}) = 2$. Likewise, you can easily construct a right inverse to the operator $\hat{L}$ and a left inverse to the operator $\hat{G}$. It still does not solve the closure problem, but can transform Eq.15 into an equivalent manner and make different regularization, see e.g., [3]. Moreover, the perturbation expansion with respect to the coupling constant $\lambda$is gaining clarity, see previous author papers.

At the end we give one more example of the operator $\hat{N}$ associated with the non-linear part of Eq.16, which may prove useful for further research in the proposed direction:

$$\hat{N} \equiv \hat{N}(\lambda_2) = \int d\tilde{x}\hat{\eta}^\star(\tilde{x})\frac{H(\tilde{x})\hat{I}}{\hat{I}-\lambda_2\hat{\eta}(\tilde{x})} + \hat{N}\hat{P}_0 \tag{28}$$

where $H(\tilde{x})$ is an arbitrary function and $\lambda_2$is a new coupling constant. This operator generates other operators appearing in Eq.15. For example, for $H = G$ and $\lambda_2 = 0$, $\hat{N} = \hat{G}$. With expansion:

$$\frac{\hat{I}}{\hat{I}-\lambda_2\hat{\eta}(\tilde{x})} = \hat{I} + \lambda_2\hat{\eta}(\tilde{x}) + (\lambda_2\hat{\eta}(\tilde{x}))^2 + \cdots$$

we get the Hurst model supplemented with the φ^3- interaction and other terms. Like in the polynomial case (Hurst model), it is easy to construct a left inverse (sic) to the operator (28):

$$\hat{N}_l^{-1} \equiv \hat{N}_l^{-1}(\lambda_2) = \int d\tilde{y}E(\tilde{y})\left(\hat{I}-\lambda_2\hat{\eta}(\tilde{y})\right)\hat{\eta}(\tilde{y}) + \hat{P}_0\hat{N}_l^{-1} \tag{29}$$

with restriction $\int d\tilde{x}E(\tilde{x})H(\tilde{x}) = 1$.

The assumption: $\hat{P}_0\hat{N} = \hat{N}_l^{-1}\hat{P}_0 = 0$ does not contradict the equality:

$$\hat{N}_l^{-1}(\lambda_2)\hat{N}(\lambda_2) = \hat{I} \tag{30}$$

6 A generalization

A generalization of the operator (28) is the formula

$$\hat{N} \equiv \hat{N}(\lambda_2) = \int d\lambda d\tilde{x} \hat{\eta}^\star(\tilde{x}) \frac{h(\lambda) H(\tilde{x}) \hat{I}}{\lambda \hat{I} - \lambda_2 \hat{\eta}(\tilde{x})} + \hat{N} \hat{P}_0 \tag{31}$$

leading to a more general expansions. A left inverse can be defined by the expression:

$$\hat{N}_l^{-1} \equiv \hat{N}_l^{-1}(\lambda_2) = \int d\mu d\tilde{y} E(\tilde{y}) \frac{h^{-1}(\mu) \hat{I}}{\mu \hat{I} - \lambda_2 \hat{\eta}(\tilde{y})} \hat{\eta}(\tilde{y}) \tag{32}$$

where the function h^{-1} is such defined that

$$\int d\mu \frac{h^{-1}(\mu) \hat{I}}{\mu \hat{I} - \lambda_2 \hat{\eta}(\tilde{y})} = \left(\int d\lambda \frac{h(\lambda) H(\tilde{x}) \hat{I}}{\lambda \hat{I} - \lambda_2 \hat{\eta}(\tilde{x})} \right)^{-1} \tag{33}$$

At such a choice of the function h^{-1} Eq.30 is satisfied and the operator $\hat{N}$ is a left invertible operator.

Another example of an operator with easy constructed left inverse is simply

$$\hat{N}(\lambda_2) = \int d\tilde{x} \hat{\eta}^\star(\tilde{x}) g\left(\lambda_2 \hat{\eta}(\tilde{x})\right) H(\tilde{x}) \tag{34}$$

In the paper [3] we started from quite different assumption, namely that the operator

$$\frac{\hat{I}}{\lambda \hat{I} - \lambda_2 \hat{\eta}(\tilde{x})} \equiv \left(\lambda \hat{I} - \lambda_2 \hat{\eta}(\tilde{x})\right)^{-1} \tag{35}$$

is a right inverse to the operator $\left(\lambda \hat{I} - \lambda_2 \hat{\eta}(\tilde{x})\right)$. This means that operator $(\lambda \hat{I} - \lambda_2 \hat{\eta}(\tilde{x}))$ is a right invertible operator. In this case, a right inverse operator (35) can be chosen as a lower triangular operator and the above property led to closing of the considered equations.

7 A possible expansion of the generating vector |V>. The perturbation principle

Let us consider Eq.15 with specified operators:

$$\left(\hat{L} + \lambda_1 \hat{N}(\lambda_2) + \hat{G}\right) |V> = \hat{P}_0 |V> + \lambda_1 \hat{P}_0 \hat{N}(\lambda_2) |V> \equiv |0>_{info} \tag{36}$$

where $\hat{L}$ is a right, and $\hat{N}$ is a left invertible operator, given by Eq.29 or Eq.32. Multiplying Eq.36 by a right inverse $\hat{L}_R^{-1}$ and using projector $\hat{P}_L = \hat{I} - \hat{L}_R^{-1} \hat{L}$,

which projects on the null space of the operator $\hat{L}$, we can rewrite the above equation in an equivalent way:

$$\left(\hat{I} + \lambda_1 \hat{L}_R^{-1} \hat{N}(\lambda_2) + \hat{L}_R^{-1} \hat{G}\right) |V> = \hat{L}_R^{-1} |0>_{info} + \hat{P}_L |V> \tag{37}$$

We will assume that solutions are symmetric:

$$|V> = \hat{S}|V> \tag{38}$$

for example, the permutation symmetric. Because the operator $\hat{L}_R^{-1}\hat{G}$ is a lower triangular operator, the Eq.37 can be equivalently transformed further as follows:

$$\left(\hat{I} + \lambda_1 \left(\hat{I} + \hat{S}\hat{L}_R^{-1}\hat{G}\right)^{-1} \hat{S}\hat{L}_R^{-1}\hat{N}(\lambda_2)\right) |V> = \\ \left(\hat{I} + \hat{S}\hat{L}_R^{-1}\hat{G}\right)^{-1} \left(\hat{S}\hat{L}_R^{-1}|0>_{info} + \hat{S}\hat{P}_L|V>\right) \tag{39}$$

where Eq.38 was used. This equation can be a starting point for the perturbation expansion with respect to the parameter λ_1 of the vector |V> satisfying the above equation.

Now let us assume that the operator $\hat{N}$ allows the following decomposition:

$$\hat{N}(\lambda_2) = \hat{N}(0) + \hat{N}_1(\lambda_2)$$

In previous works we assumed that $\hat{N}$ is a right-invertible operator and it was justified for a polynomial type of operators. Now, we assume and at least formally justify that $\hat{N}$ is the left invertible operator with a left inverse operator $\hat{N}_l^{-1}(\lambda_2)$, see Secs 5 and 6. From Eq.36, we get:

$$\left(\hat{N}_l^{-1}(\lambda_2)(\hat{L} + \hat{G}) + \lambda_1 \hat{I}\right) |V> = \hat{N}_l^{-1}(\lambda_2)|0>_{info} = 0 \tag{40}$$

We assume that

$$\hat{N}_l^{-1}(\lambda_2) = \hat{N}_l^{-1}(0) + \hat{A}(\lambda_2),\ \hat{A}(\lambda_2) \Rightarrow 0,\ for\ \lambda_2 \to 0 \tag{41}$$

The latter property makes that the terms containing the operator $\hat{A}(\lambda_2)$, for a small value of the coupling constant λ_2, will be regarded as a perturbation. Substituting (41) into Eq.40, we get:

$$\left\{\left(\hat{N}_l^{-1}(0) + \hat{A}(\lambda_2)\right)\hat{L} + \left(\hat{N}_l^{-1}(0) + \hat{A}(\lambda_2)\right)\hat{G} + \lambda_1 \hat{I}\right\} |V> = \\ \left(\hat{N}_l^{-1}(0) + \hat{A}(\lambda_2)\right) |0>_{info} \tag{42}$$

To use perturbation calculation, we have to transform the above equation further to get an equation similar to Eq.39. For this purpose, we multiply this equation by the operator $\hat{L}_R^{-1}\hat{N}(0)$. We finally get:

$$\left\{\hat{I}+\hat{L}_R^{-1}\hat{N}(0)\hat{A}(\lambda_2)\hat{L}+\hat{L}_R^{-1}\hat{N}(0)\left(\hat{N}_l^{-1}(0)+\hat{A}(\lambda_2)\right)\hat{G}+\lambda_1\hat{L}_R^{-1}\hat{N}(0)\right\}|V>$$
$$=\hat{\Pi}_L|V>+\hat{L}_R^{-1}\hat{N}(0)\left(\hat{N}_l^{-1}(0)+\hat{A}(\lambda_2)\right)|0>_{info} \tag{43}$$

where the projector $\hat{\Pi}_L=\hat{I}-\hat{L}_R^{-1}\hat{Q}_l(0)\hat{L}$ with $\hat{Q}_l(0)=\hat{N}(0)\hat{N}_l^{-1}(0)$ projects on the null space of the operator $\hat{N}_l^{-1}(0)\hat{L}$. Because the operators $\hat{L}_R^{-1}\hat{N}(0)$ and $\hat{L}_R^{-1}\hat{G}$ are lower triangular, we can further transform the above equation:

$$\left\{\hat{I}+\lambda_1\hat{L}_R^{-1}\hat{N}(0)+\hat{L}_R^{-1}\hat{N}(0)\hat{N}_l^{-1}(0)\hat{G}+\hat{L}_R^{-1}\hat{N}(0)\hat{A}(\lambda_2)\left(\hat{G}+\hat{L}\right)\right\}|V>=$$
$$\hat{\Pi}_L|V>+\hat{L}_R^{-1}\hat{N}(0)\left(\hat{N}_l^{-1}(0)+\hat{A}(\lambda_2)\right)|0>_{info} \tag{44}$$

and finally:

$$|V>\quad+\hat{S}\left(\hat{I}+\lambda_1\hat{L}_R^{-1}\hat{N}(0)+\hat{L}_R^{-1}\hat{Q}_l(0)\hat{G}\right)^{-1}\hat{L}_R^{-1}\hat{N}(0)\hat{A}(\lambda_2)\left(\hat{L}+\hat{G}\right)|V>=$$
$$\hat{S}\left(\hat{I}+\lambda_1\hat{L}_R^{-1}\hat{N}(0)+\hat{L}_R^{-1}\hat{Q}_l(0)\hat{G}\right)^{-1}\hat{\Pi}_L|V> \tag{45}$$

where symmetry (38) was also used. Here again the projector $\hat{Q}_l(0)=\hat{N}(0)\hat{N}_l^{-1}(0)$ appears.

In this approach an important element in the expansion:

$$|V>=\sum_{j=0}^{\infty}\lambda_2^j|V>^{(j)} \tag{46}$$

is the zeroth order approximation $|V>^{(0)}$. This can be calculated from Eq.36 in which $\lambda_2=0$. For (28),

$$\hat{N}(0)=\int d\tilde{x}\hat{\eta}^{\star}(\tilde{x})H(\tilde{x}) \tag{47}$$

what is a lower triangular operator with respect to projectors (13). It makes that zeroth order problem is possible to solve. Higher orders approximation are based on the Eq.45. Undetermined term, the vector $\hat{\Pi}_L|V>$, can be identified with the zeroth order approximation:

$$\hat{\Pi}_L|V>=\hat{\Pi}_L|V>^{(0)} \tag{48}$$

This equation can be called the *perturbation principle* according to which the next approximations to the zeroth order term ($\lambda_2=0$) exclusively depend on the the *perturbation operator.* In Eq.45 this is the term that contains the operator $\hat{A}(\lambda_2)$.

When Eq.36 is multiplied by the left-inverse operator $\hat{N}_l^{-1}(\lambda_2)$ - the problem arises, namely - does the obtained Eq.40 is equivalent to the initial equation?

That the answer is conditionally positive provides the following reasoning: Multiplying Eq.40 by the operator $\hat{N}(\lambda_2)$ (invertible operation), we get equation:

$$\left(\hat{Q}_l(\lambda_2)(\hat{L}+\hat{G})+\lambda_1\hat{N}(\lambda_2)\right)|V>=0 \tag{49}$$

where projector $\hat{Q}_l(\lambda_2)=\hat{N}(\lambda_2)\hat{N}_l^{-1}(\lambda_2)$. As you can see from the above, Eq.49 is not equivalent to Eq.36. But Eq.49 can be equivalently described as:

$$\left(\hat{I}+\lambda_1(\hat{L}+\hat{G})_R^{-1}\hat{N}(\lambda_2)\right)|V>=\hat{\Pi}_{L+G}(\lambda_2)|V> \tag{50}$$

where the projector $\hat{\Pi}_{L+G}(\lambda_2)=\hat{I}-(\hat{L}+\hat{G})_R^{-1}\hat{Q}_l(\lambda_2)(\hat{L}+\hat{G})$. If, however,

$$\hat{\Pi}_{L+G}(\lambda_2)|V>=\hat{P}_{L+G}|V>+(\hat{L}+\hat{G})_R^{-1}|0>_{info} \tag{51}$$

where the projector $\hat{P}_{L+G}=\hat{I}-(\hat{L}+\hat{G})_R^{-1}(\hat{L}+\hat{G})$ then Eqs (49) and (50) are equivalent to Eq.36♣.

To avoid nonphysical solutions we have to use some additional restriction like (51) and to use the perturbation principle like (48) resulting from continuity of solutions with respect to the coupling constant λ_2. In a sense we have a situation similar to that which occurs when applying the Galerkin method. The essential difference is that here any reduction strategy for Galerkin models is not at least openly used, [8].

8 The art of equations and few remarks about one side invertibility of operators and operator-valued functions

In the paper presented we used two type of operators describing basic Eq.15: the right and left invertible operators. To the right invertible operators belongs the operator $\hat{L}$ which usually describes kinematic properties of the system. An interaction of the system elements and the exterior world is described by the left invertible operators: $\hat{N}$ and $\hat{G}$. These operators here have the null spaces, responsible for the freedom of the theory, identically equal to zero. In other words, freedom of a theory is rather exclusively determined by the operator $\hat{L}$ which is the right invertible operator. It is seen from the perturbation theory and the considerations presented here. It is surprising that considered in Sec.5 examples of the operators $\hat{N}$ are able to approximate polynomial interactions, which are right invertible operators, but considered as a whole, $\hat{N}$ are left invertible. Moreovere, in addition, it naturally contains a term which reminds us the operator $\hat{G}$ describing external or background field. The mass term appears in the next approximation to the left invertible operator $\hat{N}$. Such connection of the mass term with the external field recalls the Mach's idea that the inertial mass of a body is caused by interaction with the rest of Universe.

Once again, as we can see from the above, the operators $\hat{N}$ only in polynomial approximations are the right invertible operators. It seems that left invertibility

is a natural property of operators describing interactive properties of the systems and next to such properties as symmetry, closeness, see [4], will constitute a significant limitation in the adequate theory search. On the other hand it can be assumed that the equations like (15), with the only left invertible operators, correspond to the final everything theory. However the Eq.15, in which there are right and left invertible operators, corresponds to the modern, not-complete theories needing additional information like boundary conditions to describe a particular problem. It is also interesting that if we finally would be able to describe considered, final theory, in a form of inhomogenous linear equation with a left invertible operator $\hat{A}$ and a given vector $|\Phi >$:

$$\hat{A}|V >= |\Phi > \tag{52}$$

the solution, for the vector $|V >$will be unique - in spite of a structural ambiguity of the operator $\hat{A}_l^{-1}$! For this reason, the left invertibility of the operator $\hat{A}$ is reminiscent more of both-side invertibility.

At this point we would also like to notice that expressions given in Sec.5 like (28), (31) or (32), are examples of so called (formal) *operator-valued functions* used in many areas of science. Their simplest examples are polynomial functions broadly used in quantum field theory. In this paper we have considered local operator-valued functions like (28) and (32). Formulas (28), (31) suggest their left invertibility and allows to generate a number of interactions in a uniform manner. Explicit construction of left-inverse operation also allowed to obtain compact expressions for successive approximations with respect to the minor coupling constant λ_2. The examples given in Sec.5 lead to equations in which demarcation line among primarily linear and nonlinear theory was broken, Sec.7, and perhaps, for this reason, deserve further attention.

In [3], Sec.9, we have used also local operator valued functions and applied them to closure and regularization purposes.

The main reason that the almost the same operator-valued functions $f(\hat{A})$ in [3] we treated as right invertible and here, Sec.5, we treat as left invertible was the undefined status of the functions like (28) or rather operator-valued function $\frac{\hat{I}}{\hat{I}-\lambda_2\hat{\eta}(\tilde{x})}$. Happy coincidence is that, under some additional conditions as the (49), this leads to similar results.

As additional literature we recommend: [11], [7] and [10].

9 When usual derivatives and integrals are not necessary

Since the advent of a Newton and Leibniz era derivatives and integrals are an essential tool for the description of nature. With their help, almost every equation of physics and technology is recorded. So we proceed when to their description we use the generating functionals or vectors, see [1, 10]. But tradition is not always a good guide, see the New Testament. This is what causes that in case of the generationg functionals and vectors we must not, and I believe that we

should not use derivatives, is their **formal character**. It makes no sense to talk about changes of the formal quantities. In the papers, instead derivatives and integrals we have used the operators $\hat{\eta}(\tilde{x})$ and $\hat{\eta}^{\star}(\tilde{x})$ that resemble derivatives only in the fact that they are one-side invertible operators. No Leibniz identity analoge. The canonical derivatives and integrals are used on the "micro" level of description in the paper represented by Eqs (16). To imagine the complications which can occur when on the level of "macro" we use traditional derivatives, or rather their functional analogs, we present them by means of operators $\hat{\eta}(\tilde{x})$ and $\hat{\eta}^{\star}(\tilde{x})$. We have:

$$\delta/\delta\eta(\tilde{x}) \Leftrightarrow \sum_{n=1}^{\infty}\sum_{k=0}^{n}\hat{\eta}^{\star k}\hat{\eta}(\tilde{x})\hat{\eta}^{k}\hat{P}_n \Rightarrow \hat{\eta}(\tilde{x}) \tag{53}$$

In the post-Newton-Leibniz era, in certain areas of science, much simpler variables than the canonical variables - should be used. On the other hand, used type of combination of variables reminds us a classical-quantum description of the phenomena in which, at the same time, derivatives and action integrals appear.

The above modification of the usual calculus (Newton and Leibniz (including fractional derivatives and so on)) reminds us of the transition from commutative space-time to noncommutative one. But this is only a formal similarity, because here we get rid of the generating vectors - the derivatives in the Newton and Leibniz sense, and there points and moments are got rid of the space, [13]. In other words, in our approach, at the micro-level, Newton's and Leibniz's calcules is used, and a departure from it is done at the macro-level. We believe that the micro-level is rather closer to point-spaces what it is hidden under the term: fine-grained structure. The coarse-grained structure are related to some smoothing or averaging procedures. And here comes the brand new, paradoxical phenomenon: to describe the average quantities, you must also consider correlations between them. In other words, we consider infinite collection of n-pi. If this collection of n-pi is considered in the free Fock space, the description of equations for n-pi is more effective and even can be used to describe quantum phenomena, see, e.g.,[2, 3]. This last case is probably a consequence of two facts: quantum phenomena belong to micro-level but measurments on them belong to macro-level and the free Fock space allows us to connect these two levels together. It is hoped that the same space will allow you to connect QM and GTR (sic).

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